A Machine Learning Approach to House Price Indexes

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2019-12-22

Abstract

The existing approaches for generating house price indexes (HPIs) are almost exclusively found in the realm of traditional statistical modeling. This paper offers a new approach using a machine learning model class – random forests – combined with a model-agnostic interpretability method – partial dependency – to derive an HPI. After providing an example of this approach, I then test the Interpretable Random Forest (IRF) against indexes derived from repeat sales and hedonic price models. Using data from the City of Seattle, this comparison suggests that the IRF is competitive (and occasionally superior to) popular existing methods across measures of accuracy, volatility and revision at both a city-wide and a neighborhood scale.

Introduction

Traditionally, house price indexes have been derived from highly interpretable (statistical) modeling approaches such as regression models. Both the repeat sales and the hedonic approach – the two most commonly published approaches (Hill 2012; MacGuire et al 2013) – are regression-based models. Standard statistical models are a good fit for this task as the coefficient estimates are easily convertible into standardized price indexes. House price index generation is viewed as an inferential endeavor in which the attribution of the effects of time on market movements is sought, rather than a pure prediction problem. As a result, many of the rapidly growing set of machine learning algorithms – e.g. support vector machines, random forests and neural networks – have not been used in the production of price indexes due to the fact that they do not directly and/or easily attribute price impacts to the variables or features in the model. However, with the rise of interpretability methods (Ribiero et al. 2016; Doshi-Velez and Kim 2017; Molnar 2019), these 'black-box' models can be made explainable and suitable for a more diverse set of tasks.

This paper highlights the use of partial dependence – a model-agnostic interpretability method (Molnar 2019) – to generate house price indexes from a machine learning and inherently non-interpretable model. One of the major appeals of using a model-agnostic approach is that any underlying model class could be used on the data. In this work I use a random forest, one of the more common and intuitive machine learning models. However, this choice of model class is only for convenience, as a neural network, for example, could just as easily have been used. Along with an explanation of the method and examples, the results from the application of a model-agnostic interpretability method are compared to the more traditional repeat sales and hedonic model approaches. The findings suggest that the interpretable random forest (IRF) appraoch to house price generation is competitive with (and occasionally prefereable to) the standard approaches across measure of accuracy, volatility and revision on a set of data from the city of Seattle.

The remainder of this work is organized as follows: Section two provides a brief literature review focused on applying machine learning approaches to the task of house price indexing. I then discuss the interpretable random forest (IRF) approach to creating a house price index and provide details using a dataset from the `hpiR` R package: Seattle, WA homes sales in the 2010 through 2016 period. In section four, the interpretable random forest method is compared to more traditional models across three metrics – accuracy, volatility and revision. Finally, I conclude with a discussion of the results and the reproducility of this work.

Previous Work

Since the seminal Bailey et al. (1963) study there has been considerable and sustained research effort put into comparing and improving competing methods for generating house price indexes. Published work in this sub-field of housing economics is generally focused on one or more of four aims: 1) Comparison of model differences (Case et al 1991; Crone & Voith 1992; Meese and Wallace 1997; Nagaraja et al 2014; Bourassa et al 2016); 2) Identification and correction of estimation issues or problems (Abraham & Schauman 1991; Haurin & Henderschott 1991; Clapp et al 1992; Case et al 1997; Steele & Goy 1997; Gatzlaff & Haurin 1997, 1998; Munneke & Slade 2000); 3) Creation of local or submarket indexes (Goodman 1978; Hill et al 1997; Gunterman et al 2016; Bogin et al 2019); and/or 4) Development of a new model or estimator (Case & Quigley 1991; Quigley 1995; Hill et al 1997; Englund et al. 1998, McMillen 2012; Bokhari & Geltner 2012; Bourassa et al 2016).

This work develops and tests a framework for using random forest models combined with an interpretability layer to create a house price index. The review of literature focuses on these two novel components of the work. Readers interested in a broader coverage of approaches to and issues with existing house price index methods are directed to the "Handbook on Residential Property Prices Indices"(Eurostat 2013).

Random Forests

The term ‘machine learning’ often conjures the pejorative term ‘black box’. Or rather, a model for which predictions are given but for reasons unknown and, perhaps, unknowable, by humans. For use cases where a predicted outcome or response, in itself is all that is required the ‘black box’-ness of a model or algorithm may not be an issue (Molnar 2019). However, in cases where model biases need to be diagnosed and/or individual feature or variable contributions are a key concern of the research or model application – such as for constructing house price indexes – machine learning models need to be extended with interpretabilty methods.

There are many options for the choice of machine learning model, though most all specific model classes fall into four generalized classes: 1) logical model (decision trees); 2) linear and linear combinations of trees or other features (random forests); 3) case-based reasoning (support vector machines); and 4) iterative summarization (neural networks) (Rudin and Carlson 2019). This paper uses random forests (Breiman 2001; Hastie et al 2008) as an example as they are a common modeling approach in the machine learning literature and in industry. Random forests create a large set of many decision trees, each based on a random set of the data. As each tree is grown, the partitions in the tree are limited to a random set of the variables (features) in the data. This set of (decision) trees ‘grown’ via randomness makes a random forest. To make a prediction, simply evaluate the subject instance (house in a real estate valuation context) in each tree – which gives a predicted value – and then combine all of these evaluations and take the mean (or some other measure of central tendency). The choice of the number of trees to use and the number of random variables to be considered at each partition step are (hyper) parameters that must be determined by the modeler.

Random forests, essentially bootstrapped submarketing routines, also have a natural link to real estate valuation via the selection of small subsets of like homes to drive predictions. Interestingly, random forests have been little used in academic real estate studies (see Mayer et al 2019 for an exception) and not at all in house price index creation (to the knowledge of the author). This lack of use can likely be explained by the fact that random forests are a ‘black box’ in that they do not directly create coefficient estimates as more traditional statistical models do and, therefore, do not offer a direct approach to create price indexes. A random forest model by itself will provide a predicted value but no direct explanation of how that prediction was generated. In short, they are not inherently interpretable.

Interpretability Methods

As the use of machine learning models has grown, so too have methods to help increase the interpretability of these approaches (Slack et al 2019). One such set of enhancements are termed 'model-agnostic interpretability methods' (Molnar 2019). Model agnostic interpretability methods are post-hoc models that can be applied to any learner or model in order to provide a specific enhancement or extension in the overall interpretability of the model. Model agnostic interpretability methods can fall into a number of types or classes, some of which have varying aims. Some of the most common approaches are:

- **Simulated or counter-factual scoring**. In these approaches, machine learning models compare scored (predicted) values of counter-factual observations across a given variable(s) while holding all others constant. Individual conditional expectations (ICE) (Goldstein et al 2014) and partial dependence (PD) (Friedman 2001) are standard examples of this approach. Accumulated local effects (ALE) can also be used when extensive correlations exist in the independent variables of interest (Apley 2016). Often a goal of these approaches is to understand the marginal contribution of one or more features towards the predicted value.

- **Game Theory (Shapley Values)**. A game theory or bargaining approach where players (variables or features in the model) compete to determine the optimal payout (coefficients) for their contributions to each observed price (Cohen et al 2005; Molner 2019). Shapley values, like counter-factual scoring, seek to measure marginal contribution of specific features.

- **Global and local surrogates**. Surrogate interpretable models that roughly approximate a black box model can provide human-interpretable explanations. These surrogate models can be global – spanning all observations – or local – confined to a small subset of the data, such as location. The locally interpretable model explanation (LIME) method proposed by Ribiero et al (2016b) is the most widely known local surrogate approach. Local and global surrogates are usually used to more deeply understand the prediction of one or a few individual instances.

- **Feature importance via permutation**. Judging the importance of a particular feature or variable within a black box model can be estimated via a permutation method (Gregorutti et al 2017). This approach works by estimating a baseline model with all variables as is. For each feature (variable), permute or randomize the data for that feature and re-estimate the model. Do this for all features one at a time and measure the relative degradation of model performance when each feature is randomized. This provides a (relative) measure of which variables or features are the most important. Feature importance measures are used to identify which features in the model provided the biggest (relative) gains in model performance and aid in model specification tasks.

In this work, I use measures of individual conditional expectation (ICE) and partial dependence (PD) to extract interpretable insights on real estate market behavior over time. I have chosen this approach for two primary reasons. First, the ICE/PD approach - via counter-factual scoring across the variable of interest, time – conceptually mimics the basic questions that drive real estate price indexes, namely: What would this property/house have sold for across given intervals of time, had it sold repeated? In fact, this approach does exactly that by simulating a home sale for a given property at every time period in the study (ICE) and then combines those changes in price over time across all properties (PD).

Second, ICEs and PD are one of the easiest of the above methods to compute. It should be noted, partial dependence calculations are known to be potentially biased when the variable of interest is highly correlated with other independent variables (Molnar 2019). Most variables used in standard hedonic pricing models, such as bedrooms, bathrooms and home size are often highly correlated. Fortunately, for the purposes of house price index generation the variable of interest – time of sale – is generally highly orthogonal to other control variables making partial dependence an acceptable approach. This assumption could be violated if the quality or location of housing that transacts varies greatly over time. Practically though, this is only likely to occur in a relatively small geographic area that experienced significant new construction sales. The data in our empirical tests span a large, built-out urban municipality so this concern is minimized.

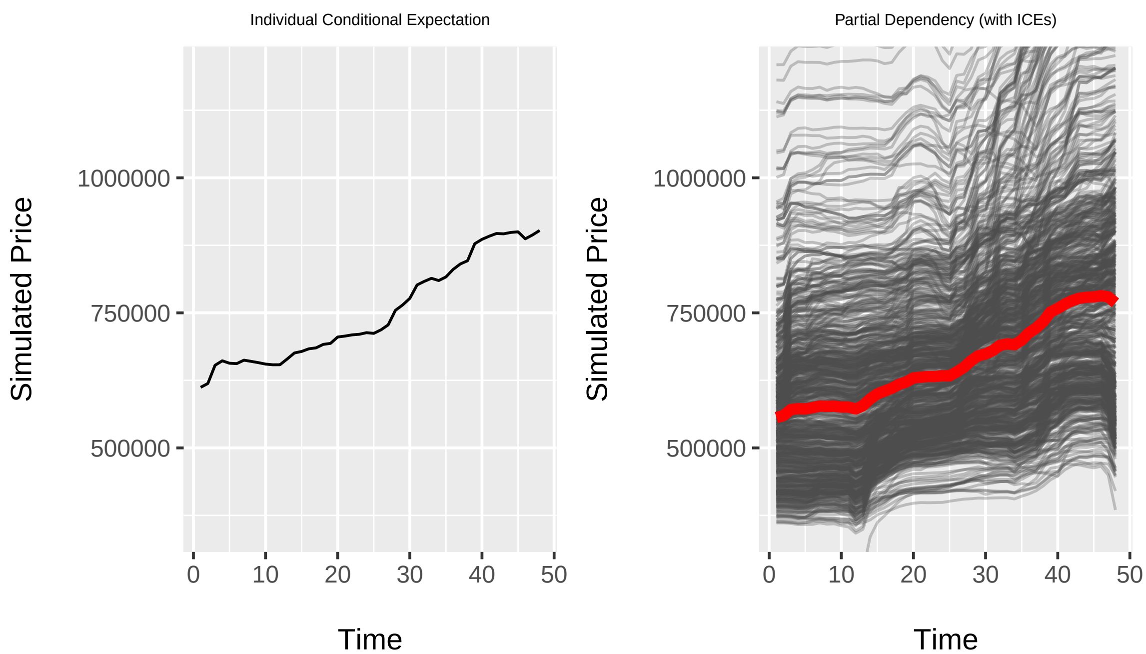
Partial dependence, and the individual conditional expectations that drive it, can be used to extract the marginal impact of each time period, conditionally, on the response or dependent variable: house prices in this case. The complexity of the resulting shape of the partial dependency – linear, monotonic, sinusoidal, spline-like, etc. – is entirely dependent on the flexibility of the underlying model being evaluated. Conceptually, an individual conditional expectation plot takes a single observation, X_i, and for one of the features or variables, X_s, simulates the predicted value of that observation under the hypothetical condition that this observation has the each individual unique value of X_s found in the entire dataset. By holding all other features constant, the marginal value of feature s on observation X_i can be simulated. This represents an Individual Conditional Expectation (ICE). Averaging across all X create a measure of partial dependency. Partial dependency is often illustrated by plotting, which is known as a partial dependency plot (Friedman 2001).

Converting this process to a real estate use for the purpose generating a house price index means valuing a given property (X_i) as if it had each unique value of time of sale (X_s) in the dataset. In other words, simulate the value of a property as if it had sold once in each time period. Do this for all properties in the dataset and average to get the full partial dependency of sale price on time of sale. A key point here is that any type or class of model could be used to simulate the series of value predictions; the approach is model agnostic.

An Example

Figure 1 illustrates example plots of an individual condition expectation (left panel) and the overall partial dependency (right) derived from a random forest model. The left hand panel applies an ICE approach on top of a random forest model with time as the variable of interest. Each point on the line, 48 in total, represents the estimated price of the example property at each month over hypothetical four-year time frame. Applying this same approach to all homes in a dataset (695 in this example), provides the thin black lines in the right hand panel. Averaging the full set of ICEs results in the partial dependency, shown in thick red line. Note that the results are expressed in raw dollar values as the partial dependency still needs to be converted to an index.

Figure 1: Example of ICE and PD Plots



Conceptual Framework

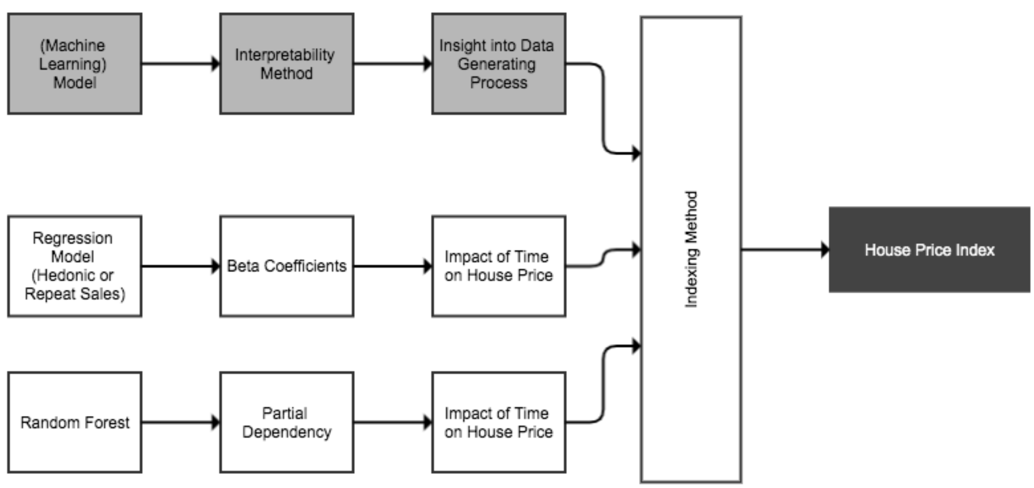
In conceptualizing how an interpretable machine learning process could map onto the standard approach(es) for creating house price indexes, it is helpful to abstract the generic process. Broadly, estimating a house price index involves the following steps:

1) Choose a **model** and apply it to the data with the purpose of explaining house prices. The chosen model will need to have a specification that accounts for one or more temporal variables or features in order to allow the model to capture or express any impacts that time may be having on prices.

2) Subject the model results to an **interpretability method** to generate insight into the data generating process (DGP). For some models this is inherent (median by time period) and for others it is a standard output (regression beta coefficients). However, the output of many machine learning models will provide only predicted values. In these cases, a post-model interpretability method will need to be applied.

3) Take the inherent or derived **insights into the DGP** – the marginal contributions of each time period to price – and convert those into an index via one of a standard set of indexes procedures.

Figure 2: Conceptual Model



More simply, this can be mapped to three decisions or steps in the process. The table below maps the three steps to actual processes from a standard hedonic price model example.

Table 1: HPI Process

Step	Description
(1) Choose a model	Specify a hedonic regression model using control variables and some configuration of temporal variables
(2) Choose an interpretability method	Extract the coefficients on the temporal variables as the marginal contribution of each time period toward observed prices in the data
(3) Choose an indexing method	Convert these coefficients to an index via the Laspreyes approach

With this framework, we can now extend the creation of house price indexes to any class of model, machine learning or otherwise, provided that a sufficient interpretability method can be applied to extract or explain the marginal impact of time period on prices. In the interpretable random forest (IRF) example above (Figure 1), the partial dependence estimates provide the 'insight into the data generating process' – the impact of time on price – that is used to generate the house price index.

Data and Model

In this section, I describe the data used in the empirical tests that follow as well as the particular model specifications employed. As part of the data discussion, I describe the geographic subsetting employed to provide local tests as well as the city-wide global analyses.

Data

The data for this study originate from the King County Assessor. All transactions of single family and townhome properties within the city of Seattle during the January 2010 through December 2016 period are included. The data are found in the `hpiR` R package and can be freely downloaded and accessed with this package. The transactions were filtered to keep only arms-length transactions based on the County's instrument, sale reason and warning codes. Additionally, any sale that sold more than once and underwent a major renovation between sales was removed as these transactions violate the constant quality assumptions made in the repeat sales models estimated below. Finally, a very small number of outlying observations – those with sales under $150,000 and over $10,000,000 were removed.

The data includes the following information for all 43,074 transactions remainder after the filtering applied above:

Table 2: Data Fields

Field Name	Type	Example	Description
pinx	chr	..0007600046	Tax assessor parcel identification number
sale_id	chr	2011..2621	Unique sale identifier
sale_price	integer	308900	Sale price
sale_date	Date	2011-02-22	Date of sale
use_type	factor	sfr	Structure type
area	factor	15	Tax assessor defined neighborhood or area
lot_sf	integer	5160	Size of lot in square feet
wfnt	binary	1	Is the property waterfront?
bldg_grade	integer	8	Structure building quality
tot_sf	integer	2200	Total finished square feet of the home
beds	integer	3	Number of bedrooms
baths	numeric	2.5	Number of bathrooms
age	integer	100	Age of home
eff_age	integer	12	Years since major renovation
longitude	numeric	-122.30254	Longitude
latitude	numeric	47.60391	Latitude

Within the data, there are 4,067 sale-resale pairs. This set of repeat transactions is limited to those which have at least a one-year span between the two sales. This constraint is applied to avoid potential home flips, which more often than not violate constant quality assumptions (Steele and Goy 1997; Clapp and Giacotto 1999).

Local Sub-samples

In addition to comparison on performance at the global (city of Seattle) level, I also break the data into the King County Assessor's 25 major residential tax assessment zones (Figure 3). Using the tax assessment zones is likely preferable to common disaggregating regions such as Zip Codes as the tax assessment zones are relatively balanced in total housing unit counts and purposefully constructed to follow local housing submarket boundaries. Of the 25 zones, 22 of them have between 1,100 and 2,300 sales over the 7-year period of this study. The remaining three have 747, 2,792 and 2,827 sales.

Figure 3: Assessment Areas and Sales

Models

Three different models are compared in this work; 1) Interpretable random forest (IRF); 2) Hedonic price (HP); and 3) Repeat sales (RS). The particular model specifications, described in detail below, remain the same across the global and 25 local geographic areas. In all cases, indexes are estimated at a monthly frequency. All models and associated metrics and visualizations are computed in the R statistical language (R Core Team 2019). Details on particular package usage are contained in the discussion on each model.

Interpretable Random Forest

Model specification for a random forest is similar to those of standard hedonic price models. The dependent variable (response) is the price of the home (logged in this case) and the independent variables (features) are those factors that are believed to explain variance in the price:

$log(P) = f(S, L, T)$

where P is the sale price, S are structural features of the home (including lot size), L are locational features and T are temporal features. More specifically, the structural features, S include home size (sq.ft.), bedroom count, bathroom count, building quality and use type (SFR or townhome), locational features, L, are latitude and longitude and the temporal feature, T, is the month of sale.

Random forest models also require parameters to control how many trees are grown, how many variables are considered at each split ("mtry") and how small each final node of the tree can be. In each case here 500 trees are grown, using an "mtry" of 3 and a minimum node (or leaf) size of 5. The `ranger` R package is used to estimate the random forest models (Wright and Ziegler 2017).

Hedonic Model

To keep the comparison as 'fair' as possible, the hedonic model uses the same set of independent variables as the random forest:

$log(P) = f(S, L, T)$

where P is the sale price, S are structural features – home size, lot size, bedrooms, baths, quality and use type – of the home, L are locational features – latitude and longitude – and T are temporal features. The temporal features in the hedonic model are treated as monthly dummy variables instead of a numeric vector as in the random forest. This allows the hedonic model to identify non-monotonic changes in prices over time – an ability that would not be possible if time were treated as an integer variable.

Following the advice of Bourassa et al (2016), I specify a robust regression to help minimize the impact of any outliers or data errors that have avoided filtering. Specifically, I use the `robustbase` R package to estimate a MM-estimator with a bi-square redescending score function (Maechler et al 2019).

Repeat Sales

Many implementations of repeat sales models implement Case and Shiller's (1989) three stage weighted approach that provides greater weight to sale pairs with shorter holding periods. Work by Steele and Goy

(1997) suggest that this may be a biasing factor as shorter holds are often less representative of standard home purchases and resales as the initial sale is more likely to be an opportune buyer. As a result of this and of work by Bourassa et al (2013), I do not weight direct by holding period length but, again, opt for a robust regression approach to help moderate any influence from outlying observation and/or changes to quality between sales that was not caught in the data preparation stage. Here, too, the `robustbase` R package is used with an MM-estimate with a bi-square redescending score function (Maechler et al 2019). The standard formulation of the repeat sales model with a logged dependent variable:

$$log(y_{it}) - log(y_{is}) = \delta_2(D_{2,it} - D_{2,is}) + ... + \delta_\tau(D_{\tau,it} - D_{\tau,is}) + u_{it} - u_{is}$$

where y_{it} is the resale, y_{is} is the initial sale and the $D_{\tau,i}$s are the temporal period dummies, -1 for the period of the first sale, 1 for the period of the second sale and 0 for all others.

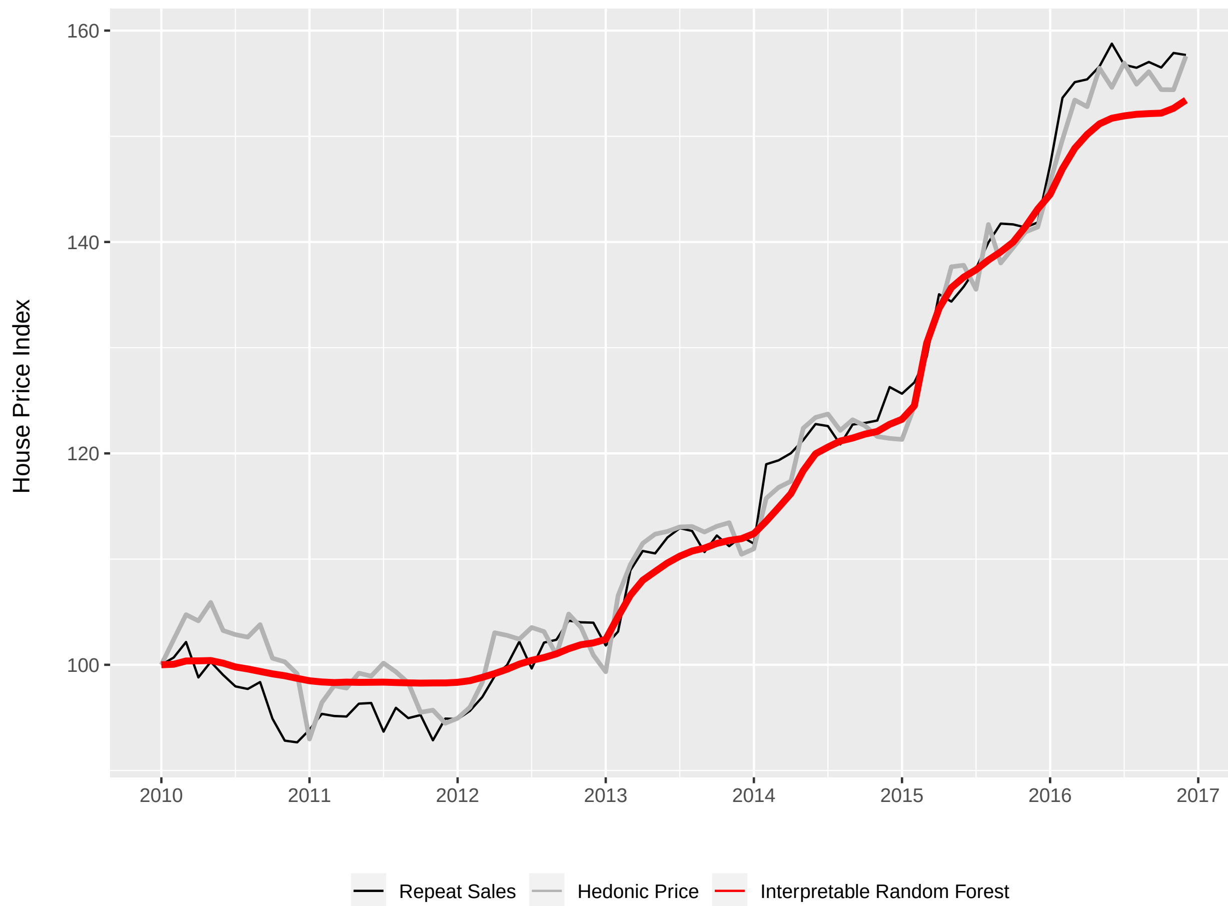
Results

I begin with a visual comparison of indexes (Figure 4) generated from the three models – interpretable random forest (IRF), hedonic price (HP) and repeat sales (RS). There are three key takeaways from this visual comparison. First, generally speaking the three models suggest similar price trends; initial evidence that the IRF approach 'works' in the sense that it can track broad market movements commensurate with established methods.

Second, the IRF method is significantly less volatile period-to-period than the HP and RS approaches. The extent of this difference and possible reasons for it will be discussed in the section on Volatility below.

Finally, the indexes diverge over the last 12 months of the index. Looking only at a single index it is difficult to pinpoint the rationale for this difference. Or rather, it is difficult to tell from this cursory visualization if this is an idiosyncratic difference due to the data in 2016 or if it is a structural difference due to the IRF's method of estimation. The fact that the random forest underlying the IRF approach treats time as a continuous variable and makes binary splits on that variable does suggest that it may produce 'overly-flat' estimates at the ends of the time period (both beginning and end) due to how the time periods are aggregated during the tree growing splits. I'll explore this hypothesis further below.

Figure 4: Comparison of Indexes



Accuracy

Overall, the indexes show similar time trends, but small variations do exist. Given the difference, is one index more accuracy than another? I test accuracy as the ability of the index to predict the second sale in a repeat sale pair. If we take the first sale in a repeat sale pair and adjust it with the index, how close is this adjusted value to the actual price of the second sale? When measuring the error – or the difference between predicted and actual second sale – I use log metrics due to their ability to avoid denominator bias and the skewness in possible error metrics that results (Tofallis 2015). The formula for calculating errors is:

$$Error_i = log(price_{i,pred}) - log(price_{i,actual})$$

where the $price_{i,pred}$ is the time adjusted prediction and $price_{i,actual}$ is the actual sale price of the repeat (second) transaction in the sales pair.

Using repeat sales as our validation criteria does impart some comparative advantage to the repeat sales (RS) method; however, accuracy is evaluated in two distinct ways that lessen any perceived advantage this may provide. First, I evaluate the ability of indexes to predict prices out-of-sample using a k-fold approach. In this case, I use a 10-fold approach, whereby 90% of the sample is used to create an index that is then used to predict repeat sale prices on the 10% that was held out of the sample. This is done for each 10% random holdout. By doing so, no evaluation observations (repeat sales) can influence the index used to predict its second sale price. I refer to errors from this approach as 'K-Fold' errors.

The convention k-fold approach can be a bit problematic in a longitudinal setting such as estimating house price indexes. In any of the 90/10 splits, most of the holdout set is being valued by an index that 'knows the future'; or one that was estimated with information well after the holdout observation occurred. This situation does not well approximate many of the actual use cases of house price indexes which are often most interested in the fidelity of the index at the most recent point in time (i.e. last month or quarter). Another approach to out-of-sample measurement is to forward predict, or to predict 'out-of-time'. As an example, to measure out-of-time accuracy on a sale-resale pair that sold in period 1 and then again in period 30, we would use the data from periods 1 to 29 to create an index, then forward-cast the index one period and evaluate the forward indexed price from period 1 against the subsequent sale in period 30. In other words, we want to ensure the model is ignorant of the validation point (the period 30 sale) as well as an other future knowledge of market trends as they are highly correlated within a market area across time. One major downfall of this approach is that is requires specifying and implementing a forecasting approach, which itself adds additional uncertainty to the process. The one-period forward predictions are made with the R `forecast` package, using a simple exponential smoother with additive errors and no trend or seasonality (type 'ANN' (Hyndman et al 2008)). I refer to these error as 'Forecast' errors.

As both accuracy metrics have downfalls, I measure them both with an eye towards understanding if there are relative differences in performance based on the error metric chosen. I review these implications in the Discussion section below. Each error metric is evaluated at both the global and local scales. Additionally, the metrics examine median absolute percentage error (MdAPE) and median percentage error (MdPE). MdAPE measures the accuracy of the index while MdPE measures its bias.

Global Accuracy

Global accuracy result are shown in Table 3. For the k-fold metrics, the interpretable random forest approach shows the best accuracy (MdAPE), though the repeat sales model results in the lowest bias (lower MdPE). Moving over to forecast evaluation, the hedonic price approach is the clear winner in terms of accuracy, with the repeat sales model again showing the lowest bias. The IRF model performs relatively poorly in the forecast approach compared to k-fold. As alluded to above, this could be a direct product of the splitting processes used in random forest model which is likely to be slow to keep up with rapidly increasing markets (as experienced over this time frame).

Based on the global accuracy values, neither of the three index methods clearly outperforms the others. Repeat sales do hold lower biases in this sample, but, as is often the case, at the expensive of some bit of accuracy. Additionally, the reduced bias of the repeat sales model could also be a direct result of using repeat sales as the validation set.

Table 3: Global Accuracy

Model	MdAPE (k-fold)	MdPE (k-fold)	MdAPE (forecast)	MdPE (forecast)
Repeat Sales	0.0770	-0.0019	0.0806	-0.0064
Hedonic Price	0.0796	-0.0283	0.0781	-0.0343
Interpretable Random Forest	0.0756	-0.0275	0.0853	-0.0588

Local Accuracy

Examining the local accuracy numbers shows some marked change from the Global figures (Table 4). The much smaller sample sizes in the assessment zones creates a considerable decrease in accuracy (errors +70%) for the RS model across both the k-fold and forecast metrics. Accuracy numbers for the HP and IRF also increased, but very slightly so. In terms of accuracy, the IRF approach remains the most accurate in the K-Fold scenario, while hedonic pricing again dominates in a forecasting situation. Biases remain high in the IRF approach. Despite the large degradation in accuracy, the RS model remains unbiased in the K-Fold, but not in the forecast evaluation framework.

Table 4: Local Accuracy

Model	MdAPE (k-fold)	MdPE (k-fold)	MdAPE (forecast)	MdPE (forecast)
Repeat Sales	0.1367	-0.0028	0.1513	-0.0477
Hedonic Price	0.0846	-0.0137	0.0821	-0.0328
Interpretable Random Forest	0.0812	-0.0410	0.0961	-0.0823

From the global and local accuracy analyses, three general findings result:

- The move from global to local models did not improve accuracy in any model, and greatly harmed the repeat sales models.
- Interpretable random forest models are more accurate than hedonic pricing in k-fold, but this relationship switches in a forecast scenario.
- All models are either unbiased (repeat sales) or show bias on the low end – are under-predicting second sale prices

I spend the remainder of this paper exploring each of these findings through additional measures of model performance.

Volatility

Volatility measures the variation in the index value from period to period. While there is no ideal minimal level of volatility that is desired; no volatility at all signifies a perfectly flat index which is not desirable if there are market movements. In general lower volatility is usually preferred to higher. High volatility may be a sign of over-fitting from the underlying model, or, as is often the case in areas with few sales, simply a product of small sample sizes.

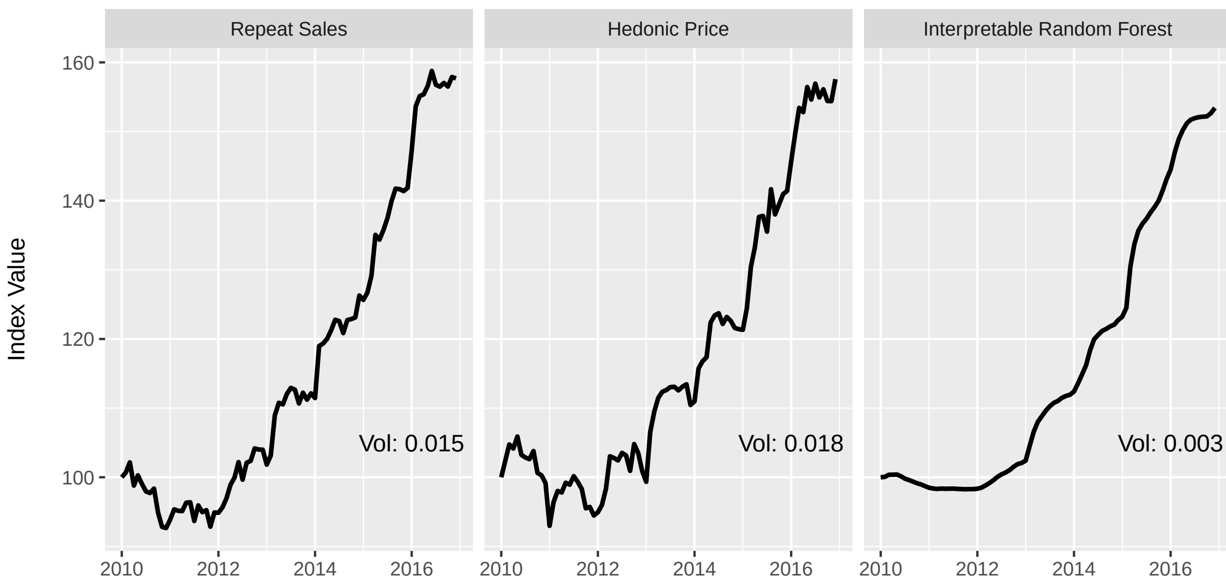
What is desirable is an index that tracks the market without fluctuating widely above and below the actual trend each period. In this paper, volatility is measured as the standard deviations of period-to-period changes in a rolling four-period time span.

$V = sd(D_{t,t+1,t+2})$ where $D = index_k - index_{k-1}$

This is an appealing metric as consistent, monotonic changes over a four month span – the three measures of period changes – will produce very low standard deviations. On the contrary, wildly fluctuating indexes with irregular directionally movements will produce high volatility measures.

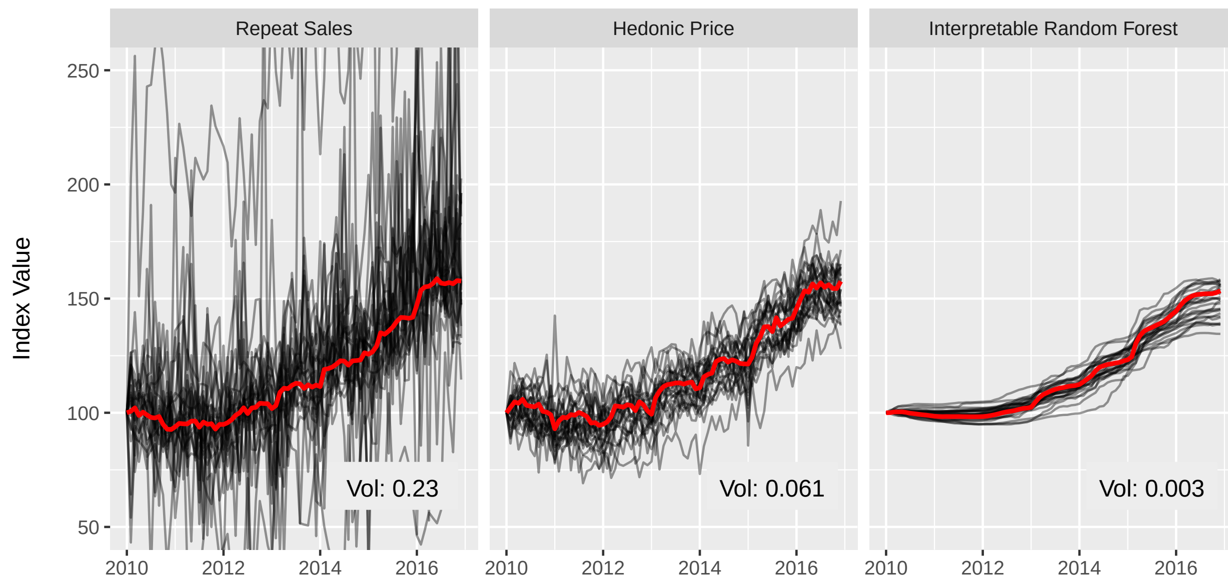
Using the volatility metric defined above, we can measure the differences in 'smoothness' of the three approaches (Figure 5). While the repeat sales and hedonic price methods have differences in their period to period movements, overall their levels of volatility are similar. The volatility of the IRF model is 1/5 to 1/6 that of the others, confirming what we saw visually in Figure 4.

Figure 5: Global Volatility



Computing volatility for each of the 25 local areas (assessment zones) shows a much wider differential in volatility among the methods (Figure 6). Moving to local models creates vastly greater volatility for the repeat sales model. The smaller sample size has a much greater impact on the RS model as this approach already is an inefficient use of data, as it does not use all transactions. The HP model shows about 3 times the volatility at the local level – on average – than globally. For the IRF model, the local models are just as smooth as the global approach, a marked contrast from the other two methods. Additionally, the spread of index values for the local areas are much tighter for the IRF than the others.

Figure 6: Local Volatility



Revision

The final metric, revision, is the amount that previous index values change as a new period is added to the index. For example, imagine we have 30 periods worth of data and we create an index with this data and the 5th period in our index is estimated at 110. We then receive data for period 31, we re-calculate the entire index and the index value for period 5 is revised up to 111 due to changes in our model's coefficients as a result of the additional observations. This is a revision of 1.

Within this work I measure period-wise revision as the individual mean of the revisions for each period in the index as it expands out to cover the entire time period. The revision for period k is:

$R_k = \sum (K_j - K_{j-1})/j$

where j is the new index being generated after each addition of data.

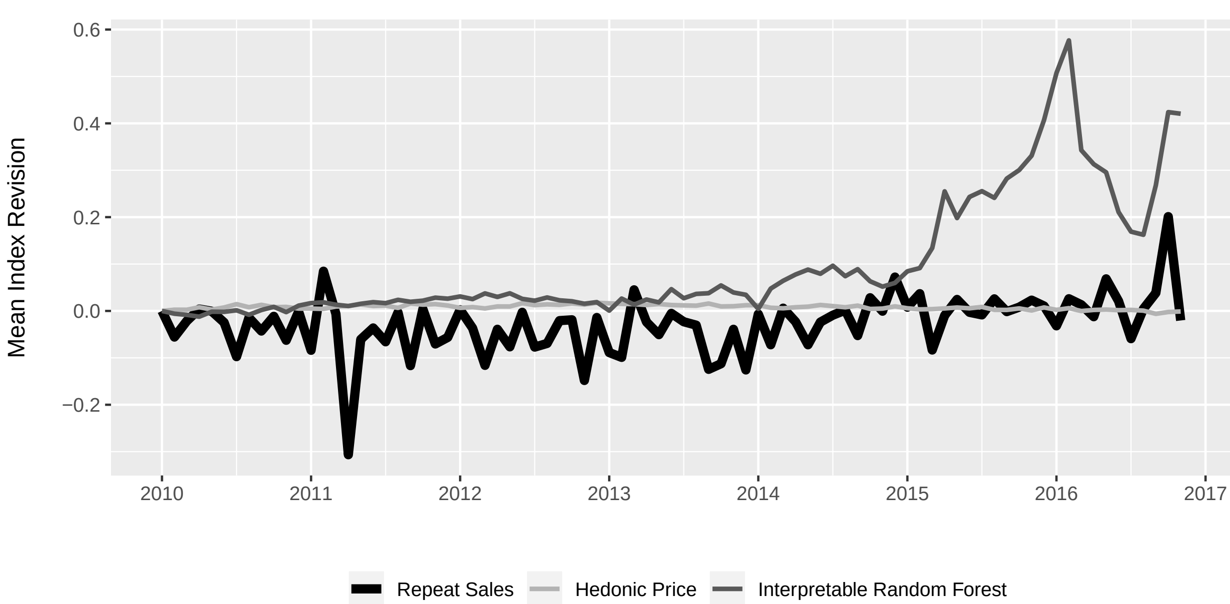
In the index analyzed below, I begin each with a 24-month period of training data to create the first index. I then add in data from period 25 and recalculate the index, measuring the revision for periods 1 to 24. The same is then done for period 26 (measuring revision for periods 1 to 25) and on up through period 84. There is no revision number for period 84 as it is only estimated once.

Revisions are particularly interesting and worthwhile metric for house price use cases that are continuously updating an index over time. Large and consistent re-statements of prior index periods can be problematic, especially if the indexes are used to back financial instruments (Deng and Quigley 2008). For indexes used to make adjustments to training data for automated valuation models and other appraisal purposes, systematic revisions in same direction – sustained downward adjustments over time, for example – can result in biased property valuation estimates and other propagated errors.

Figure 7 shows the mean revision amount across the time period, by each model for the global models. There are three key takeaways here: 1) The repeat sales model has higher revisions in the earlier periods (final period aside) and these revisions are usually downward; 2) The hedonic model has almost no revisions over time; and 3) The interpretable random forest approach suffers from the highest revisions in the later periods, all of which are upwards.

The findings regarding the stability of the hedonic model and the tendency for downward adjustments from the repeat sales approach, echo those of previous work (Clapp and Giacotto 1999; Clapham et al 2006). The later period adjustment for the IRF model mimic earlier findings that suggest that the random forest tends to lag in periods of high price growth (the latter part of the study period).

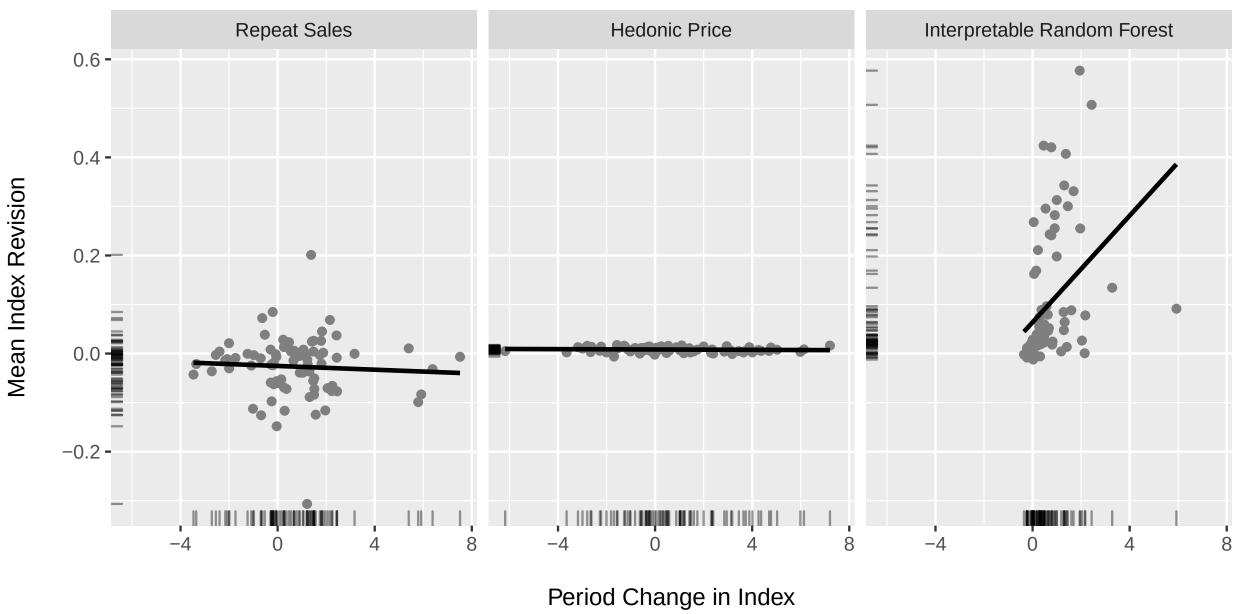
Figure 7: Mean Revision by Period (Global)



To further explore the relationship between overall price movements and adjustments, Figure 8 plots the mean revision in each period (y-axis) against the percent change in prices during that period. For the repeat sales model we see slightly greater downwards revisions in periods of high appreciation, though the trend is rather weak. This may suggest that 'flip' type sales are a greater cause of revision than the underlying market, similar to rationale provided by Clapp and Giacotto (1999).

As there are no real revisions for the hedonic model (middle panel) we see no noteworthy trends here. At odds with repeat sales, the IRF approach shows significant positive correlation between revisions and the underlying price changes during the period. Again, this supports the idea that the random forest lags during period of high appreciation, only to 'catch up' later via upward adjustments.

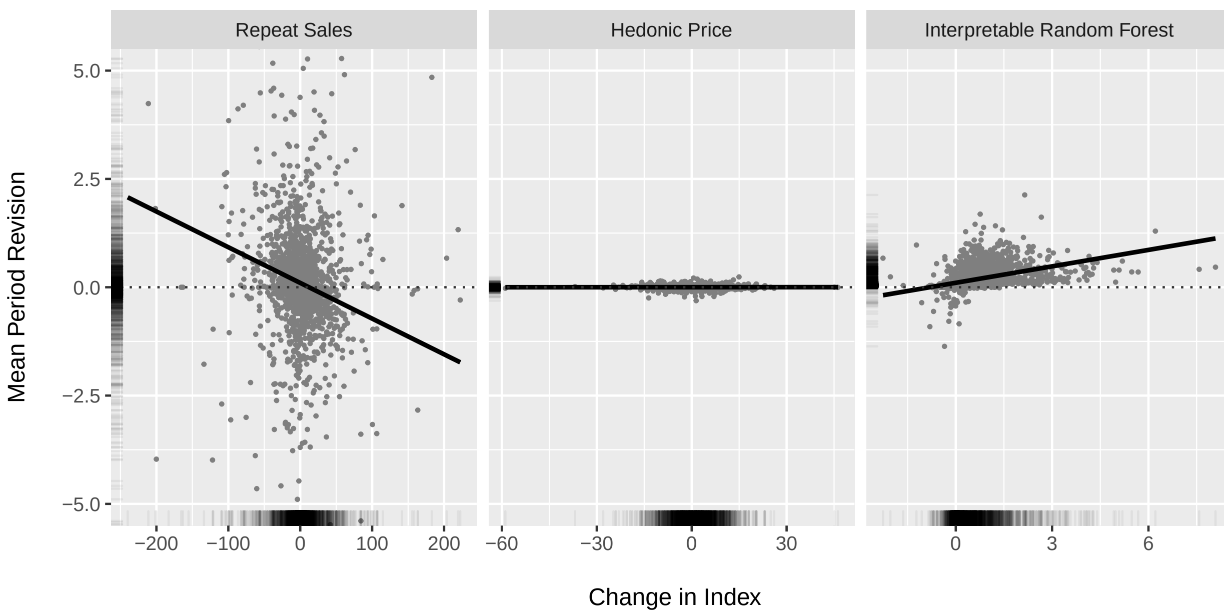
Figure 8: Revision vs Price Movement (Global)



Making the same comparisons – revision vs period index change – for the local indexes reveals a number of findings. One note for interpreting these plots (Figure 9): the X-axes vary between plots in order to fully capture the differences in volatility between the different models, the Y-axes are stable.

First, the hedonic approach (middle panel) remains very stable even with the greatly reduced sample sizes in the local models.The repeat sales approach shows a much more pronounced negative relationship between price movements and revisions at the local scale. In other words, the higher the underlying price movement, the greater the downward revisions that occur over time. Again, for the IRF model the opposite is true. Higher price appreciation means higher upward revisions. Nearly all revisions to the local IRF models are upwards, but this fact should be not be interpreted as suggesting IRF doesn't generally adjust downwards. Rather, this study period is one of either flat or increasing home prices (as evidenced by the X-axis in the right hand panel) and the lack of downward revisions is likely a result of the the sample period and the method itself.

Figure 9: Revision vs Price Movement (Local)



Discussion

This paper introduces a new approach to estimating a house price index through the use of an interpretable machine learning model. More specifically, it applies a model agnostic interpretability method – partial dependency – to an inherently opaque machine learning technique – a random forest. From this combination we are able to extract the change in prices over time, holding quality constant. This approach is particularly well suited conceptually for a house price index as it values individual properties once each period to create an index; essentially the purpose of a house price index. The index itself tracks very similarly to those produced by more traditional repeat sales and hedonic price approaches supporting the viability of this approach empirically as well.

The interpretable random forest (IRF) is then compared against the two traditional methods for measures of accuracy, volatility and revision; both globally and across 25 smaller assessment regions in the city of Seattle. There are a number of major findings.

In terms of accuracy at the global scale, all three models are relatively close with IRF the most accurate in a k-fold validation and the hedonic model in a forecast environment. The repeat sales model remains the most un-biased, though this may be a product of repeat sales being used as the accuracy validation set. Additional research is required to develop a more standardized approach for evaluating house price index accuracy. At the local scale, both the IRF and hedonic price methods again show solid accuracy performance in contrast to the repeat sales models which degrade considerably at the local level.

Index smoothness is where the IRF method proves particularly adept. Volatility for the IRF model ranges from 1/6 of the others at the global to upwards of 1/75th at the local scale. Given that the accuracy metrics for the IRF are comparable, in terms of producing a smooth index with high fidelity to the market movements, the IRF approach offers a marked improvement.

Like the repeat sales approach, the IRF method suffers from revision as new data is added each period. The scale of the revisions are slightly greater than the repeat sales at a global scale, though considerably smaller, relatively, when the models are run locally. Notably, the repeat sales model revises down during periods of appreciation while the IRF revises up during these periods of price growth. For the repeat sales this is generally caused by data issues (short holds lacking constant quality Steele and Goy (1997)), while for the IRF model it is likely a derivative of the random forest algorithm itself. The revisions to price movement correlations are amplified when moving from the global to the local model.

Overall, from an accuracy and a volatility perspective the IRF model is a viable replacement for the repeat sales model and is competitive with a hedonic price approach depending on whether or not accuracy or volatility are the preferred metrics. A deficiency of the IRF approach is the systematic, positively correlated revisions during periods of rapid market movements; it is here that the 'smoothness' of the resulting IRF models becomes a liability. More work it needed in this direction.

More broadly, this work highlights the viability of an inherently uninterpretable machine learning model (black box) matched with a model agnostic interpretability method to derive house price indexes. While this work paired random forest with a partial dependency analysis, there is no reason other models – such as a neural network – could not be combined with other interpretability methods – such as Shapley values. As discussed in the Conceptual Framework section, all that is needed to create a house price index is a model of house prices, a method for understanding the data generating process in respect to the impact of time on price and an indexing method. The traditional repeat sales and hedonic price approaches are entrenched mostly out of convenience – they offer directly interpretable beta coefficients – and not, necessarily, out of any underlying superiority. As machine learning approaches and their interpretability methods continue to develop, the status quo here will likely be continually challenged. This is the first work to offer both a general framework for conceptualizing house price indexes in a machine learning context as well as offer a direct test of one approach across a number of important metrics – accuracy, volatility and revision.

Reproducibility and Software

This work is completely reproducible. All raw data, code and general instructions to exactly recreate the analyses above is found at https://www.github.com/anonymousreauthor/irf_house_price_index[1] All code is written in the R statistical language. In addition to the `hpiR` package, which includes the custom functions for the IRF models and the wrapper functions that make for easy computation of accuracy, volatility and revision figures this work also directly uses the following R packages: `caret`(Kuhn 2019), `dplyr`(Wickham et al 2019), `forecast`(Hyndman et al 2019), `ggplot`(Wickham 2016), `imputeTS`(Moritz and Bartz-Beielstein 2017), `knitr`(Xie 2019), `lubridate`(Grolemund and Wickham 2011), `pdp`(Greenwell 2017), `purrr`(Henry and Wickham 2019), `ranger`(Wright and Ziegler 2017), `robustbase`(Maechler et al 2019), `tidyr`(Wickham and Henry 2019) and `zoo`(Zeileis and Grothendieck 2005).

[1]NOTE to reviewers: This will be switched to my actual Github Repository after blind peer review.

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